PS 110 - Answers

1a) Convert everything to the same units
   and then keep track of the presence of uncertainty:

   If you convert to grams
   \[
   \begin{align*}
   1.380 & \quad \text{g} \\
   2.346 & \quad \text{g} \\
   1.387 & \quad \text{g} \\
   0.796 & \quad \text{g} \\
   \hline
   1616.8432 & \quad \text{g}
   \end{align*}
   \]

   There is the 1st position of uncertainty.

   You keep only 1 uncertain digit, so you report:

   \[1620 \text{ g} \implies \text{better is } 1.62 \text{ kg}\]

   or \[1.62 \times 10^3 \text{ g} \text{ if you want units in grams}\]

1b) Same ideas here:

   \[1.6168 \quad \text{kg} \]
   \[-1.274 \quad \text{kg} \]

   \[0.344 \quad \text{kg}\]

   Thus, \[0.34 \text{ kg}\]

   \[\text{you can gain or lose sig. fig. when you add and/or subtract.}\]

1c) \[
\left(38.71 \times 10^{-3} \text{ L}\right) \left(0.398 \frac{\text{mol}}{\text{L}}\right) \left(148.92 \frac{\text{g}}{\text{mol}}\right) \left(\frac{1}{0.9864 \text{ g}}\right)
\]

   \[= 2.33 \text{ (no units)}\]

   OR

   \[
   \left(38.71 \text{ mL}\right) \left(0.398 \frac{\text{mmol}}{\text{mL}}\right) \left(148.92 \frac{\text{mg}}{\text{mmol}}\right) \left(\frac{1}{986.4 \text{ mg}}\right)
   \]

   \[= 2.33 \text{ (no units)}\]
2. a) Two possibilities here when the ruler is used.

Poss. 1 If the ruler is graduated in tenths of inches, it may be possible to estimate between the markings, so you could get a value with uncertainty in the hundredths place. Then you'd have to estimate the uncertainty.

Ex: a part of a ruler graduated in tenths of inches

Suppose a measurement of something was made here. You could call this 1.7 for sure, and then guess the next place — this could be 1.75 but the "5" is pretty shaky (maybe by 0.02). So uncertainty is 0.02 inch.

Poss. 2 Just stick with the measured graduations on the ruler and report the reading as either 1.7 or 1.8 inch, in which case the uncertainty would be 0.1 inch.

The same kind of arguments apply when the micrometer is used. The uncertainty will be either 0.000 — (with some estimate at the last place) — some digit here or 0.001 in. (if you report to the nearest graduation mark).

b) If you use the tenths' ruler and estimate the hundredth's place with an absolute uncertainty of 0.02 in., you'd probably read the diameter as 0.75 in.

Then, the \% Rel. Uncert. = \frac{0.02 \text{ in.}}{0.75 \text{ in.}} \approx 2.6 \approx 3%.

If you use the micrometer, even if you can't estimate a fourth digit, your absolute uncertainty would be no worse than 0.001 in. You'd read the diameter as 0.754 in.

Then, the \% Rel. Uncert. = \frac{0.001}{0.754} \times 100 \approx 0.13 \approx 0.1%.

Since the second case has the lower \% Rel. Uncert., it is the "better" (higher quality) measurement.
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3. a) Rank the measurements: \( n = 9 \)

\[ 0.85, 0.94, 0.95, 0.99, 1.01, 1.05, 1.05, 1.05, 1.06 \]

Only choice for outliers are the "end" meas.
Clearly, 0.85 is a more suspicious meas. than 1.06 — check it first.

\[ Q = \frac{|0.85 - 0.94|}{1.06 - 0.85} = \frac{0.09}{0.21} = 0.43 \]

This is less than the critical value for \( n = 9 \) (0.506 in \( Q \) table), so 0.85 is not an outlier.

(If 0.85 isn’t, then clearly 1.06 won’t be either.)
So use 0.85 and 1.06 in all calcs.

b) \( \bar{y} = \frac{\sum y}{n} = \frac{8.95}{9} = 0.994 \quad \text{0.99 \% Mn} \)

c) Median = middle value when ranked = 1.01 \% Mn

d) \( S = (0.21)(0.34) = 0.071 \% \text{ Mn} \)

\( k = 0.34 \) (when \( n = 9 \))

e) \[ S = \sqrt{\frac{\sum y^2 - (\sum y)^2}{n}} = \sqrt{\frac{8.9399 - (8.95)^2}{9}} = 0.070 \% \text{ Mn} \]

f) For Range case
\[ \% \text{ RSD} = \left( \frac{0.071 \% \text{ Mn}}{0.99 \% \text{ Mn}} \right) \times 100 = 7.2 \% \]

For S-S case
\[ \% \text{ RSD} = \left( \frac{0.070 \% \text{ Mn}}{0.99 \% \text{ Mn}} \right) \times 100 = 7.17 \% \]
Using the SS value for standard deviation

Conf. Interval, C.I. = \( \frac{t \cdot s}{\sqrt{n}} \)

For \( n = 9 \) and 95% confidence, \( t = 2.306 \) (in Student t table)

\[
C.I. = \frac{(2.306)(0.070)}{\sqrt{9}} = 0.0538 = 0.054\% \text{ Mn}
\]

95% Confidence limits are

\( 0.99 \pm 0.054 = 0.94\% \text{ Mn} \text{ to } 1.04\% \text{ Mn} \)

4. Should check for outliers first. \( n = 4 \)

17.55, 17.86, 18.02, 18.11

Worst suspect:

\[
Q = \frac{17.86 - 17.55}{18.11 - 17.55} = 0.31 \div 0.56 = 0.55
\]

In the Q-table, \( Q_{\text{critical}} = 0.829 \). Thus, no outliers.

a) Taking the mean as best estimate,

\[
\bar{y} = \frac{\sum y}{n} = \frac{71.54}{4} = 17.88\% \text{ S0}_4^{2-}
\]

(If median were used, best estimate would be

\[
(17.86 + 18.02) = 17.94\% \text{ S0}_4^{2-}
\]

b) By Range method

\[
S = (0.49)(0.56) = 0.27\% \text{ S0}_4^{2-}
\]

By SS method

\[
S = \sqrt{\frac{1279.6746 - (71.54)^2/4}{3}} = 0.25\% \text{ S0}_4^{2-}
\]

Do not round off any of these values until the final square root has been calculated.

%RSD (Range) = \( \frac{27}{17.88} \times 100 = 1.5\% \)

%RSD (SS) = \( \frac{25}{17.88} \times 100 = 1.4\% \)
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5. Outliers? \( n = 9 \)

\[ 68.14, 68.33, 68.39, 68.42, 68.55, 68.68, 68.77, 68.81, 68.98 \]

the measurement at the end farthest from its neighbor should be tested

Range = \( 68.98 - 68.14 = 0.84 \)

\[ Q = \frac{68.33 - 68.14}{0.84} = 0.19 = 0.23 \]

\[ Q_{\text{critical}} = 0.506 \] (for \( n = 9 \) in a table) — so no outliers here

Best estimate of \( \% \text{NaCl} \) in bag?

Mean = \( \frac{\sum y_i}{n} = 68.56 \% \text{NaCl} \)

Sample std dev = \( \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}} = 0.2681 \% \text{NaCl} \)

95\% C.I. value = \( \frac{0.0206}{(2.306)(0.2681)} = 0.21 \% \text{NaCl} \)

So (with 95\% assurance) the true \( \% \text{NaCl} \) in the bag could range anywhere from \( 68.56 - 0.21 = 68.34 \% \text{NaCl} \) to \( 68.56 + 0.21 = 68.77 \% \text{NaCl} \)

Since the Decision Limit (68.75 \% NaCl) is included in this interval, the difference between our best estimate (68.56) and the "Rule" (68.75) could have been caused solely by random error. (This would be the case 95\% of the time.)

We conclude there IS NO DIFFERENCE between our best estimate and the "real" value and accept the bag as having the proper \% NaCl in it.